

# A Novel Hybrid Artificial Bee Colony Algorithm for Numerical Function Optimization

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**Abstract**— Artificial Bee Colony (ABC) algorithm is very interesting population based swarm optimization technique. This technique is motivated by means of extraordinary nature of honey bees. ABC algorithm commonly used to get to the bottom of nonlinear and complex problems. Comparable to other population based strategies, ABC also has some negative aspects. It is computationally steep because of its sluggish temperament during search process. This paper planned a new approach so as to enhance performance of ABC strategy by combining positive characteristics two most popular modification of ABC algorithm named Gbest-guided ABC (GABC) and Memetic search in ABC (MeABC). It combines best part of both algorithm and balance process of exploration and exploitation in ABC. GABC improves exploitation and MeABC improves exploration. The proposed algorithm is named as Memetic search with Global best solution in ABC (MGABC). It is tested over some impartial test problems with diverse intricacy; results indicate that the anticipated algorithm outperforms the original ABC, GABC and MeABC in most of the experiments. It is also tested over two real world problems namely Welded beam design problem and compression spring problem and results are very impressive for both problems.

**Keywords**— Particle Swarm Optimization, Artificial Bee Colony Algorithm, Swarm Intelligence, Evolutionary Computation, Memetic Algorithm.

## I. INTRODUCTION

Nature Inspired Algorithms (NIAs) mimics the intelligent behaviour of social insects like bees, ants, termites, fish and birds etc. Swarm Intelligence getting popularity now days and become a rising and fascinating area. It depends on the cooperative behaviour of societal living thing. Societal individual make use of their skill of societal wisdom to crack multifaceted everyday jobs. The main power of swarm based optimization strategy is multiple interactions in societal colonies. Swarm intelligence strategies have potential to solve complex factual world optimization problems as the preceding study [1, 2, 3, 4] have exposed. The ant colony optimization (ACO) [1], bacterial foraging optimization (BFO) [5], particle swarm optimization (PSO) [2] and artificial bee colony algorithm [6] are some of popular algorithms that have surfaced in recent years.

Recently D. Karaboga proposed a very simple and easy to implement strategy motivated by extra ordinary food foraging behaviour of honey bee insects and named it as artificial bee colony algorithm [6]. Reminiscent of other population based optimization algorithms, this algorithm also has a population of budding solutions. Food source for

a honey bee represent one possible solution. Fitness of a particular food source computes its quality that represent amount of nectar in a food source. Performance of ABC algorithm depends on steadiness between searching of local search space and utilization of best feasible outcomes. Sometimes it is observed that the ABC stops proceeding headed for the global optimum despite the fact that the local optimum not achieved [7]. Some research revealed that the position update equation for ABC technique is fine at exploration however it is not good at exploitation [8]. For that reason, it is exceedingly vital to expand a local search strategy in the fundamental ABC in order to take advantage of the search space so that balance between intensification and diversification can be maintained. Therefore this work proposed a hybrid approach by combining best properties of two approaches GABC [9] and MeABC [10]. “Gbest-guided artificial bee colony (GABC) algorithm for numerical function optimization” proposed by G. Zhu et al.[9] is good in exploitation and “Memetic search in artificial bee colony (MeABC) algorithm” projected by J.C. Bansal et al.[10] is good in exploration of local search space. The proposed strategy integrates capability of both GABC and MeABC and results in a new algorithm named as memetic search with global best solution in ABC (MGABC). Additionally, the planned algorithm is evaluated on some un-biased benchmark problems and experimental outcome are judged against GABC, MeABC and basic ABC.

Remaining paper is planned in the subsequent manner: Section 2 gives concise summary of the basic ABC. Section 3 recent modifications in ABC algorithm are detailed. Memetic Search with Global best solution in ABC (MGABC) is wished-for and results are shown in Section 4. In next Section, an all-inclusive set of experimental outcome are presented. To finish, in Sect. 6, paper is concluded followed by references.

## II. ARTIFICIAL BEE COLONY ALGORITHM

The ABC algorithm that is motivated by extraordinary food foraging conduct of honey bee insects is very simple to understand and implement. Each food source for honey bee symbolizes solution of a particular problem in ABC algorithm. Fitness of a particular food source computes its quality that represent amount of nectar in a food source. In ABC algorithm, honey bees are categorized into three sets that is to say employed bees, onlooker bees and scout bees. The employed bees and the onlooker bees must be same in quantity. The employed bee search new food sources and gather information concerning the eminence of the food

sources. Some bees wait in the beehive and observe the activities of employed bees. Based on the activities of employed bees they select food sources are identified as onlooker bees. When a food source rejected due to low quality, then they are replaced by new food sources randomly. The ABC strategy follows iterative process it repeats these three phase again and again. Each of the phases is illustrated as follows:

*A. Phases of ABC Algorithm*

The ABC algorithm follows three key segments while deciding solution [6]:

- First phase is to propel the employed bees on the food sources, modernize position of food sources based on quality of particular food source;
- In second phase onlooker bees select a food source with higher probability based on its fitness.
- Third phase engender randomly fresh food sources in place of unwanted food sources.

*1) Initialization of Swarm*

Total amount of food sources also known as population, the number of trial subsequent to which a food source is considered as rejected also known as limit and the termination condition also known as maximum number of cycle are three important parameter in ABC algorithm. D. Karaboga [6] suggested that the quantity of food sources should be identical to the employed bees or onlooker bees. At the time of initialization it is considered that food sources (SN) are evenly dealt swarm, where a D-dimensional vector represent each food source  $x_i$  ( $i = 1, 2 \dots SN$ ). Each food source is initialized using Eq. (1) [6]:

$$x_{ij} = x_{\min j} + rand[0,1](x_{\max j} - x_{\min j}) \tag{1}$$

Where

- $rand[0,1]$  is a function that engender an equally dispersed arbitrary numeral in range  $[0,1]$ .

*2) Employed Bee Phase*

The position of current solution modernized with the help of knowledge of individual’s understanding and the appropriateness of the recently established solutions. Existing food sources replaced with innovative food source having superior fitness value. The location of  $j^{th}$  dimension of  $i^{th}$  candidate modernizes using Eq. (2) [6]:

$$v_{ij} = x_{ij} + \phi(x_{ij} - x_{kj}) \tag{2}$$

Where

- $X_{ij}-X_{kj}$  decide size of step,
- $k \in \{1, 2, \dots, SN\}, j \in \{1, 2, \dots, D\}$  are two indices that are haphazardly preferred in such a way that  $k \neq i$  in order to make sure that step size has some pinpointing enhancement.

*3) Onlooker Bee Phase*

The counting of onlooker bees is identical to the quantity of employed bees. During this segment all employed bee share quality of novel food sources through onlooker bees in form of fitness. Every food source judged based on its probability of selection. The highly fitted solution gets elected by the onlooker. There are various techniques for

calculation of probability; however it must be a function of fitness. Probability of selection for each food source is determined with its fitness as per Eq. (3) [6]:

$$P_{ij} = \frac{fit_i}{\sum_{i=1}^{SN} fit_i} \tag{3}$$

*4) Scout Bee Phase*

In case when the position of a particular food source is not modernized for a threshold (in term of number of cycles), that food source is derelict and a new phase starts named scout bees phase. The bees that are allied in the midst of the deserted food source transformed into scout bee and the food source is substituted by the capriciously elected food source within the exploration space. New food sources generated using Eq. (4) [6].

$$x_{ij} = x_{\min j} + rand[0,1](x_{\max j} - x_{\min j}) \tag{4}$$

Algorithm 1 outlines major steps of ABC:

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**Algorithm 1:** Artificial Bee Colony Algorithm

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Initialize all parameters;

Reiterate while annihilation criteria is not meet

Step 1: Employed bee phase for computing new food sources.

Step 2: Onlooker bees phase for modernizing position the food sources based on their quantity of nectar.

Step 3: Scout bee phase for probing new food sources in place of discarded food sources.

Step 4: Memorize the finest food source known up to now.

End of while

Output: The finest solution recognized up to now.

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**III. RECENT MODIFICATIONS IN ABC ALGORITHM**

Recently a number of researchers are working on ABC algorithm to solve optimization problems. N.K. Garg et al. [11] proposed Gbest-ABC to solve load flow problem in place of Newton-Raphson method. M. K. Apalak et al. [12] carried out the study of the layer optimization designed to make best use of the lowest elementary frequency of proportioned laminated complex plates subjected to some grouping of the three conventional boundary conditions, and then applied the ABC algorithm to the layer optimization. S. Kumar et al. [13] proposed a crossover based ABC algorithm with an additional phase borrowed from genetic algorithm, improved onlooker bee phase [14], randomized memetic ABC [15], an improved memetic search in ABC [16] with local search phase inspired by golden section search process. S. Pandey et al. [17] planned a customized ABC algorithm and applied it to solve travelling salesman problem. H. Sharma et al. [18] planned a hybrid of opposition based learning and levy flight local search with basic ABC algorithm. J. C. Bansal et al. [19] anticipated a balanced ABC so as to balance exploration and exploitation process. A comparative study carried out by S. Kumar et al. [20] shows the cons and pros of different hybrids of ABC algorithm. A. Kumar [21] proposed a fitness based position update in ABC by introducing concept that highly fitted solution are more feasible in comparison to low fitted solution. S. Kumar et al proposed a new local search strategy in ABC [22] and enhanced local search in ABC [23] motivated by golden section search.

P.A. Moscato [24] described a new category of stochastic global exploration strategy hybrid with evolutionary strategies named as memetic algorithms (MA). The concept of MAs is motivated by Universal Darwinism. "Universal Darwinism" recommends that advancement is not elite to natural systems; i.e., it is not limited to the perspective of the genes only, but also pertinent to all multifaceted organism that demonstrates the ideology of inheritance, variation and selection, therefore satisfying the qualities of an embryonic system. The MAs are motivated by both the Darwinian principles of natural evolution and Dawkins' notion of a meme. Generally it is known as Memetic Algorithm or Memetic Computing. Recently M Liu et al. [25] proposed an improved neighbourhood search and a memetic algorithm hybrid with random key crossover to solve capacitated Arc routing problem. MS Pishvaei et al. [26] designed a bi-objective logistic network with the help of memetic algorithm. SU Ngueveu et al. [27] anticipated a novel memetic algorithm to resolve the problem of cumulative capacitated vehicle routing. Y Nagata et al. [28] proposed a penalty based edge assembly memetic algorithm to address the problem of vehicle routing. A number of researchers used memetic algorithms to solve complex problems that belong to class of NP problems.

Kang et al. [29] proposed a memetic algorithm named HJABC by integrating Hooke Jeeves [30] local search strategy in ABC algorithm. HJABC is a hybrid of both escalation search motivated by the Hooke Jeeves pattern search and the basic ABC. HJABC modify the method of fitness ( $fit_i$ ) computation and integrates the Hooke-Jeeves local search in basic ABC. HJABC includes amalgamation of investigative move and pattern move to look for best possible outcome of problem. The first, step exploratory move think about one variable at a moment so as to choose opposite route of exploration process. The subsequent stride is pattern search to accelerate search in decisive way by exploratory budget.

J. C. Bansal et al [10] proposed a memetic search in ABC aggravated by Golden Section Search (GSS) method [31]. In MeABC only the superlative particle of the recent swarm brings up to date itself in its propinquity. G. Zhu et al. [9] proposed Gbest-guided artificial bee colony (GABC) algorithm for numerical function optimization that is good in exploitation. GABC algorithm modifies solution search equation illustrated by Eq. (5) as follow:

$$v_{ij} = x_{ij} + \phi_{ij}(x_{ij} - x_{kj}) + \Psi_{ij}(y_j - x_{ij}) \quad (5)$$

Here  $\phi_{ij}$  is random number in range [0, 1],  $\Psi_{ij}$  is a capricious number in period [0, C], for some positive constant C. In Eq. (5) third term in the right-hand side is a new added expression called gbest term,  $y_j$  is the  $j^{\text{th}}$  component of the global best solution.

#### IV. MEMETIC SEARCH WITH GLOBAL BEST SOLUTION IN ABC (MGABC) ALGORITHM

There are two basic distinctiveness of the population-based optimization algorithms such as PSO (2), BFO (5),

GA (32) and DE (33) as well as ABC [6] known as exploration and exploitation. In above optimization techniques, the capability to explore the different indefinite regions in the solution space to determine the global optimum is known as exploration. The exploitation is the capability to utilize the acquaintance of the prior high-quality solutions to come across enhanced solutions. Both the exploration and exploitation are opposing conception, and consecutively to bring about superior optimization performance, these two abilities must be balanced.

D Karaboga et al. [7] studied a number of modifications of ABC algorithm for global optimization and bring forth that the ABC demonstrates deprived performance and leftovers ineffectual at some stage in the exploration of the search space. In ABC algorithm, some feasible solution brings up to date itself using the information given by an arbitrarily chosen feasible solution in the existing swarm. J. C. Bansal et al. [10] also compared performance of ABC and its memetic variants and conclude that it lack in balance between intensification and diversification. It is observed that step size in position update equation play a very critical role while searching for optimal solutions. If step size is very large then there are chances that it may skip optimal solution and a tiny step size may cause a problem of stagnation. This step size also depends on a random number  $\phi_{ij} \in [-1, 1]$ . Quality of solutions depends on this step size. If  $\phi_{ij}$  has large value then step size may too large and small value of  $\phi_{ij}$  cause very small step size then the convergence rate of ABC may notably diminish as it takes additional time to move towards most favourable value.

An appropriate sense of balance in this extent of step is able to balance the searching and utilization potential of the ABC at the same time. However, in view of the fact that this extent of step consists of arbitrary component therefore the balance cannot be done by hand. Some local search algorithms are merged with ABC algorithm in order to improve exploitation competence. For that reason, this paper establish, memetic search with Gbest-guided ABC to balance the diversity and speed of convergence for ABC. It redefines the search range of two parameters in GSS process and applies GSS based search process in ABC just after scout bee phase, additionally solution update equation modified as shown in Eq. (5) inspired by GABC [9] to take benefit of knowledge about the global best solution to lead the exploration of entrant solution. A new parameter  $\Psi_{ij}$  introduced in position update equation. Here  $\Psi_{ij}$  is a uniform arbitrary number in range [0, C], wherever C is a positive constant. Equation (5) shows that the innovative entrant solutions budge towards the global best solution due to Gbest term; for that reason, the adapted solution exploration equation illustrated by Eq. (5) can amplify the utilization of ABC algorithm. If value of C is 0 then Eq. (5) will become same as Eq. (2) or it works as basic ABC algorithm. When one increase value of C from 0 to a particular value then exploitation capability of ABC algorithm will increase due to Eq. (5). The value of C must not be too large as in that case it may skip true solutions.

TABLE I. TEST PROBLEMS FOR MGABC ALGORITHM

Test Problem	Objective Function	Search Range	Optimum Value	D	Acceptable Error
Griewank	$f_1(x) = \frac{1}{4000} \left( \sum_{i=1}^D (x_i^2) \right) - \left( \prod_{i=1}^D \cos\left(\frac{x_i}{\sqrt{i}}\right) \right) + 1$	[-600, 600]	$f(0) = 0$	30	1.0E-05
Rastrigin	$f_2(x) = \sum_{i=1}^D [x_i^2 - 10 \cos(2\pi x_i) + 10]$	[-5.12, 5.12]	$f(0) = 0$	30	1.0E-05
Alpine	$f_3(x) = \sum_{i=1}^n  x_i \sin x_i + 0.1x_i $	[-10, 10]	$f(0) = 0$	30	1.0E-05
Salomon Problem	$f_4(x) = 1 - \cos(2\pi \sqrt{\sum_{i=1}^D x_i^2}) + 0.1(\sqrt{\sum_{i=1}^D x_i^2})$	[-100, 100]	$f(0) = 0$	30	1.0E-01
Inverted Cosine wave function	$f_5(x) = -\sum_{i=1}^{D-1} \left( \frac{\exp\left(\frac{-(x_i^2 + x_{i+1}^2 + 0.5x_i x_{i+1})}{8}\right)}{\cos(4\sqrt{x_i^2 + x_{i+1}^2 + 0.5x_i x_{i+1}})} \right)$ Where $I = \cos(4\sqrt{x_i^2 + x_{i+1}^2 + 0.5x_i x_{i+1}})$	[-5, 5]	$f(0) = -D+1$	10	1.0E-05
Neumaier 3 Problem (NF3)	$f_6(x) = \sum_{i=1}^D (x_i - 1)^2 - \sum_{i=2}^D x_i x_{i-1}$	[-100, 100]	$f(0) = -210$	10	1.0E-01
Beale function	$f_7(x) = (1.5 - x_1(1 - x_2))^2 + (2.25 - x_1(1 - x_2^2))^2 + (2.625 - x_1(1 - x_2^3))^2$	[-4.5, 4.5]	$f(3, 0.5) = 0$	2	1.0E-05
Colville function	$f_8(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 + 90(x_4 - x_3^2)^2 + (1 - x_3)^2 + 10.1[(x_2 - 1)^2 + (x_4 - 1)^2] + 19.8(x_2 - 1)(x_4 - 1)$	[-10, 10]	$f(1) = 0$	4	1.0E-05
Kowalik function	$f_9(x) = \sum_{i=1}^{11} \left( a_i - \frac{x_i(b_i^2 + b_j x_2)}{b_i^2 + b_j x_3 + x_4} \right)^2$	[-5, 5]	$f(0.1928, 0.1908, 0.1231, 0.1357) = 3.07E-04$	4	1.0E-05
Shifted Rosenbrock	$f_{10}(x) = \sum_{i=1}^{D-1} (100(z_i^2 - z_{i+1})^2 + (z_i - 1)^2) + f_{bias}$ $z = x - o + 1, x = [x_1, x_2, \dots, x_D], o = [o_1, o_2, \dots, o_D]$	[-100, 100]	$f(o) = f_{bias} = 390$	10	1.0E-01
Shifted Griewank	$f_{11}(x) = \sum_{i=1}^D \frac{z_i^2}{4000} - \prod_{i=1}^D \cos\left(\frac{z_i}{\sqrt{i}}\right) + 1 + f_{bias}$ $z = (x - o), x = [x_1, x_2, \dots, x_D], o = [o_1, o_2, \dots, o_D]$	[-600, 600]	$f(o) = f_{bias} = -180$	10	1.0E-05
Hosaki Problem	$f_{12}(x) = (1 - 8x_1 + 7x_1^2 - \frac{7}{3}x_1^3 + \frac{1}{4}x_1^4)x_2^2 \exp(-x_2)$	$x_1 \in [0, 5], x_2 \in [0, 6]$	-2.3458	2	1.0E-06
Meyer and Roth Problem	$f_{13}(x) = \sum_{i=1}^5 \left( \frac{x_1 x_3 t_i}{1 + x_1 t_i + x_2 v_i} - y_i \right)^2$	[-10, 10]	$f(3.13, 15.16, 0.78) = 0.4E-04$	3	1.0E-03
Sinusoidal	$f_{14}(x) = -[A \prod_{i=1}^n \sin(x_i - z) + \prod_{i=1}^n \sin(B(x_i - z))]$ $A = 2.5, B = 5, z = 30$	[-10, 10]	$f(90+z) = -(A+1)$	10	1.00E-02

The proposed MGABC algorithm also adds an additional local search phase just after scout bee phase motivated by golden section search [31]. Here this algorithm use modified golden section search process. It modifies the process of calculation of function  $f_1$  and  $f_2$ . The original golden section search process [10] compute function  $f_1$  and  $f_2$  using Eq. (6) and (7).

$$f_1 = b - (b - a) \times \psi \tag{6}$$

$$f_2 = a + (b - a) \times \psi \tag{7}$$

Where

- $\Psi$  represent golden ratio.

Equation (6) and (7) replaced by Eq. (8) and (9) by adding a new parameter in each equation. Value of these newly introduced parameters taken in such a way after a sequence of experiments that the local search process will explore maximum search space. The modified golden section search process use Eq. (8) and (9).

$$f_1 = \phi_1 (b - (b - a) \times \psi), \text{ where } \phi_1 \in [0, 1] \tag{8}$$

$$f_2 = \phi_2 (b - (b - a) \times \psi), \text{ where } \phi_2 \in [-1, 0] \tag{9}$$

**Algorithm 2:** Modified Golden Section Search Process

Initialize a= -1.2 and b = 1.2  
 Repeat while (|a-b| < ε)  
 Compute  $f_1 = \phi_1 * (b - (b-a) * \Psi)$ , Where  $\phi_1 \in [0, 1]$   
 $f_2 = \phi_2 * (a + (b-a) * \Psi)$ , Where  $\phi_2 \in [-1, 0]$   
 Generate two new solutions  $X_{new1}$  and  $X_{new2}$  using  $f_1$  and  $f_2$  respectively according to MeABC  
 Calculate  $f(X_{new1})$  and  $f(X_{new2})$  for objective function  
 if ( $f(X_{new1}) < f(X_{new2})$ ) then  $b = f_2$   
     if ( $f(X_{new1}) < f(X_{best})$ )  
         then  $X_{best} = X_{new1}$   
 else  $a = f_1$   
     if ( $f(X_{new2}) < f(X_{best})$ )  
         then  $X_{best} = X_{new2}$

The newly introduced local search phase (As shown in algorithm 2) improve exploration process as Memetic search in artificial bee colony (MeABC) algorithm is good in exploration of local search space and modified solution update strategy improve exploitation as Gbest-guided artificial bee colony (GABC) algorithm for numerical function optimization that is good in exploitation. For that reason, in these modifications, enhanced solutions get more chance in search process and minimize the hazard of fewer steadinesses here. The memetic search with global best solution in ABC is outlined in algorithm 3 as follow:

**Algorithm 3:** Memetic search with global best solution in ABC

Initialize all parameters;  
 Repeat while Termination criteria is not meet  
     Step 1: Employed bee phase for compute new food sources.  
     Step 2: Onlooker bees phase for updating position the food sources based on their amount of nectar using equation (5).  
     Step 3: Scout bee phase for searching new food sources in place of abandoned food sources.  
     Step 4: Apply modified golden section search using algorithm 2.  
 End of while  
 Output: The best solution recognized so far.

**V. EXPERIMENTAL RESULTS**

*A. Considered Test Problem*

Artificial Bee Colony algorithm with modifications in position update equation and an additional step applied to the fourteen benchmark functions for whether it gives improved outcome or not at diverse probability and also applied for two factual world problems namely compression spring problem and welded beam design problem. Benchmark problems considered in this paper are of different individuality like uni-model or multi-model and separable or non-separable and of diverse dimensions. So as to analyse the performance of MGABC it is applied to global optimization problems ( $f_1$  to  $f_{14}$ ) outlined in Table I. Test problems  $f_1$  -  $f_{14}$  are taken from [34][35].

**Welded beam design optimization problem ( $f_{15}$ ):** It is a problem of designing a welded beam with minimum cost [36]. Here it is required to identify the minimum cost of fabricating for the welded beam subject to restraints on bending stress  $\sigma$ , load of buckling  $P_c$ , end deflection  $\delta$ , shear stress  $\tau$ , and side constraint. In case of this problem four design variables are considered:  $x_1, x_2, x_3$  and  $x_4$ . The

simple mathematical formulation of the objective function is described as follows:

$$f_{15}(x) = 1.1047x_1^2x_2 + 0.04822x_3x_4(14.0 + x_2)$$

Subject to:

$$g_1(x) = \tau(x) - \tau_{max} \leq 0, g_2(x) = \sigma(x) - \sigma_{max} \leq 0,$$

$$g_3(x) = x_1 - x_4 \leq 0, g_4(x) = \delta(x) - \delta_{max} \leq 0,$$

$$g_5(x) = P - P_c(x) \leq 0$$

$$0.125 \leq x_1 \leq 5, 0.1 \leq x_2, x_3 \leq 10 \text{ and } 0.1 \leq x_4 \leq 5$$

Where

$$\tau(x) = \sqrt{\tau'^2 - \tau' \tau'' \frac{x_2}{R} + \tau''^2}, \tau' = \frac{P}{\sqrt{2x_1x_2}}, \tau'' = \frac{MR}{J},$$

$$M = P(L + \frac{x_2}{2}), R = \sqrt{\frac{x_2^2}{4} + (\frac{x_1 + x_2}{2})^2}, \sigma(x) = \frac{6PL}{x_4x_3^2},$$

$$J = \frac{2}{\sqrt{2}x_1x_2[\frac{x_2^2}{4} + (\frac{x_1 + x_2}{2})^2]}, \delta(x) = \frac{6PL^3}{Ex_4x_3^2},$$

$$P_c(x) = \frac{4.013Ex_3x_4^3}{6L^2} (1 - \frac{x_3}{2L} \sqrt{\frac{E}{4G}})$$

$$P=6000 \text{ lb, } L=14 \text{ in, } \delta_{max} = 0.25 \text{ in, } \sigma_{max} = 30000 \text{ psi,}$$

$$\tau_{max} = 13600 \text{ psi, } E = 30 \cdot 10^6 \text{ psi, } G = 12 \cdot 10^6 \text{ psi}$$

The finest recognized solution is (0.205730, 3.470489, 9.036624, 0.205729), which gives the function value 1.724852. Acceptable error for this problem is 1.0E-01.

**Compression Spring ( $f_{16}$ ):** The compression spring problem [10] diminishes the weight of a compression spring that is subjected to constraints of shear stress, surge frequency, minimum deflection and restrictions on exterior diameter and on design variables. In case of compression spring three design variables considered: The diameter of wire ( $x_1$ ), mean coil diameter ( $x_2$ ) and count of active coils ( $x_3$ ). Simple mathematical representation of this problem is:

$$x_1 \in \{1, 2, 3, \dots, 70\} \text{ granularity}_1$$

$$x_2 \in [0.6; 3], x_3 \in [0.207; 0.5] \text{ granularity}_{0.001}$$

And four constraints

$$g_1 := \frac{8c_f F_{max} x_2}{\pi x_3^3} - S \leq 0, g_2 := l_f - l_{max} \leq 0$$

$$g_3 := \sigma_p - \sigma_{pm} \leq 0, g_4 := \sigma_w - \frac{F_m a x - F_p}{K} \leq 0$$

$$\text{Where : } c_f = 1 + 0.75 \frac{x_3}{x_2 - x_3} + 0.615 \frac{x_3}{x_2}, F_{max} = 1000,$$

$$S = 189000, l_f = \frac{F_{max}}{K} + 1.05(x_1 + 2)x_3, l_{max} = 14, \sigma_p = \frac{F_p}{K},$$

$$\sigma_{pm} = 6, F_p = 300, K = 11.5 \times 10^6 \frac{x_3^4}{8x_1x_2^2}, \sigma_w = 1.25$$

And the function to be minimized is

$$f_{16}(X) = \pi^2 \frac{x_2 x_3^2 (x_1 + 2)}{4}$$

TABLE II. COMPARISON OF RESULTS FOR TEST PROBLEMS USING MGABC ALGORITHM

Test Problem	Algorithm	MFV	SD	ME	AFE	SR
Griewank	ABC	5.95E-03	1.01E-02	5.95E-03	82628	60
	MeABC	5.25E-04	2.29E-03	5.25E-04	68278.54	95
	GABC	3.76E-04	1.82E-03	3.76E-04	32414.86	96
	MGABC	4.01E-04	1.73E-03	4.01E-04	43920.57	94
Rastrigin	ABC	3.51E+00	1.34E+00	3.51E+00	99682.5	1
	MeABC	5.06E-01	6.05E-01	5.06E-01	97742.3	21
	GABC	5.68E-06	3.02E-06	5.68E-06	33930	100
	MGABC	8.62E-06	1.34E-06	8.62E-06	38949.7	100
Alpine	ABC	1.93E-02	1.42E-02	1.93E-02	100000	0
	MeABC	6.88E-03	5.15E-03	6.88E-03	100048	0
	GABC	8.44E-06	4.21E-06	8.44E-06	55460.5	98
	MGABC	8.90E-06	4.52E-06	8.90E-06	71398.86	94
Salomon Problem	ABC	1.58E+00	2.55E-01	1.58E+00	100000.8	0
	MeABC	9.26E-01	3.21E-02	9.26E-01	25024.57	100
	GABC	9.56E-01	5.27E-02	9.56E-01	73490.53	75
	MGABC	9.21E-01	3.16E-02	9.21E-01	<b>18486.73</b>	100
Inverted Cosine wave function	ABC	-2.36E+00	4.38E-01	6.64E+00	100009	0
	MeABC	-8.86E+00	3.34E-01	1.42E-01	71506.64	67
	GABC	-9.00E+00	6.37E-04	7.11E-05	42517.61	99
	MGABC	-8.97E+00	1.59E-01	2.64E-02	41630.4	96
Neumaier 3 Problem(NF3)	ABC	-5.57E+01	2.55E+01	1.54E+02	100039.45	0
	MeABC	-2.10E+02	1.30E-02	8.60E-02	24141.75	100
	GABC	-2.06E+02	4.76E+00	3.68E+00	99877.91	1
	MGABC	-2.10E+02	1.02E-02	8.92E-02	24954.08	100
Beale function	ABC	8.72E-06	1.20E-06	8.72E-06	16548.79	100
	MeABC	4.78E-06	2.88E-06	4.78E-06	5886.5	100
	GABC	5.46E-06	2.80E-06	5.46E-06	10149.98	100
	MGABC	4.95E-06	3.06E-06	4.95E-06	<b>2950.63</b>	<b>100</b>
Colville function	ABC	2.52E-01	1.81E-01	2.52E-01	100030.75	0
	MeABC	2.01E-02	2.63E-02	2.01E-02	69601.61	49
	GABC	3.62E-02	3.41E-02	3.62E-02	92990.18	21
	MGABC	7.17E-03	2.75E-03	7.17E-03	29332.22	<b>97</b>
Kowalik function	ABC	4.80E-04	6.83E-05	1.72E-04	91125.07	21
	MeABC	4.16E-04	4.98E-05	1.08E-04	60627	76
	GABC	4.37E-04	1.06E-04	1.29E-04	73865.34	63
	MGABC	3.95E-04	2.15E-05	8.73E-05	44895.28	<b>98</b>
Shifted Rosenbrock	ABC	3.96E+02	8.62E+00	6.29E+00	95356.5	8
	MeABC	3.95E+02	7.37E+00	4.81E+00	91459.08	13
	GABC	3.90E+02	6.88E-01	3.96E-01	82180.65	43
	MGABC	3.91E+02	1.89E+00	7.01E-01	78044.88	42
Shifted Griewank	ABC	-9.20E+01	1.57E+01	8.80E+01	100008.45	0
	MeABC	-1.80E+02	3.89E-03	2.13E-03	64040.77	74
	GABC	-1.80E+02	1.61E-03	3.83E-04	36835.53	93
	MGABC	-1.80E+02	1.87E-03	4.74E-04	41641.71	<b>94</b>
Hosaki Problem	ABC	-2.32E+00	2.45E-02	2.85E-02	100021.38	0
	MeABC	-2.35E+00	6.29E-06	5.99E-06	16484.45	84
	GABC	-2.35E+00	6.66E-06	5.83E-06	7344.23	93
	MGABC	-2.35E+00	6.31E-06	6.39E-06	19308.27	81
Meyer and Roth Problem	ABC	1.91E-03	5.33E-06	1.95E-03	25153.05	95
	MeABC	1.91E-03	3.07E-06	1.95E-03	11708.15	100
	GABC	1.90E-03	2.91E-06	1.94E-03	4623.66	100
	MGABC	1.91E-03	2.97E-06	1.95E-03	4768.71	100
Sinusoidal	ABC	-5.57E-01	2.08E-01	2.94E+00	100034.07	0
	MeABC	-3.49E+00	2.88E-03	8.87E-03	69696.99	82
	GABC	-3.49E+00	2.77E-03	7.93E-03	46185.34	96
	MGABC	-3.49E+00	4.04E-03	8.49E-03	36723.71	95
Welded Beam Design Problem	ABC	2.07E+00	1.37E-01	3.43E-01	98939.6	2
	MeABC	1.91E+00	9.08E-02	1.87E-01	91297.94	14
	GABC	1.84E+00	2.66E-02	1.15E-01	85242.94	34
	MGABC	1.83E+00	1.80E-02	1.03E-01	61077.76	<b>70</b>
Compression Spring Problem	ABC	2.65E+00	1.10E-02	2.40E-02	97302.16	5
	MeABC	2.64E+00	1.22E-02	1.32E-02	93453.69	15
	GABC	2.64E+00	1.34E-02	1.76E-02	93796.42	11
	MGABC	2.63E+00	1.04E-02	8.43E-03	89339.11	<b>23</b>

The best ever identified solution is (7, 1.386599591, 0.292), which gives the fitness value  $f=2.6254$  and  $1.0E-04$  is tolerable error for compression spring problem.

**B. Experimental Setting**

The proposed memetic search with global best solution in ABC algorithm is tested over above discussed standard problems and compared with original ABC algorithm [6], memetic search in ABC [10] and Gbest guided ABC [9] in order to check its performance, following experimental setting is considered:

- The size of colony= Population size SN =50
- Number of Employed bee = Number of Onlooker bee =SN/2 = 25
- The maximum number of cycles for foraging MCN = 5000
- Number of repetition of experiment =Runtime =100
- Limit =1500, A food source which could not be enhanced in the course of "limit" trial is discarded by its employed bee.

The mean function values (MFV), standard deviation (SD), mean error (ME), average function evaluation (AFE) and success rate (SR) of considered problem have been recorded.

Experimental setting for ABC, MeABC and GABC are same as MGABC.

TABLE III SUMMARY OF TABLE II OUTCOME

Test Problem	ABC vs MGABC	MeABC vs MGABC	GABC vs MGABC
Griewank	+	-	-
Rastrigin	+	+	-
Alpine	+	+	-
Salomon Problem	+	+	+
Inverted Cosine wave function	+	+	-
Neumaier 3 Problem	+	-	+
Beale function	+	+	+
Colville function	+	+	+
Kowalik function	+	+	+
Shifted Rosenbrock	+	+	-
Shifted Griewank	+	+	+
Hosaki Problem	+	+	-
Meyer and Roth Problem	+	+	-
Sinusoidal	+	+	-
Welded Beam Design Problem	+	+	+
Compression Spring Problem	+	+	+
<b>Total number of + sign</b>	<b>16</b>	<b>14</b>	<b>8</b>

**C. Result Comparison**

The experimental results of MGABC with above said setting are sketched in Table II. This table illustrates that a good number of the times MGABC outperforms in terms of competence (with fewer number of function evaluations) and consistency as contrast to additional painstaking algorithms. The planned technique constantly gets better AFE and more often than not it also gets better SD and ME. It is due to two new modifications in basic ABC. Table III have review of table II results. Here, ‘+’ sign point out that the MGABC is superior to the considered approaches and ‘-

’ sign specify that the proposed technique is not so good or the variation is minute.

Further, an evaluation is made on the basis of speed of convergence for the measured strategies by computing the AFEs. If AFEs are less it means algorithm has higher rate of convergence. As the ABC algorithm is stochastic in nature and golden section search also has random parameters. So as to minimize the consequence of this stochastic nature the reported AFEs for all test problems are averaged over 100 runs. Acceleration Rate (AR) computed in order to check rate of convergence. Acceleration Rate (AR) is defined as follows, based on the AFEs for the two algorithms ALGO and MGABC:

$$AR = AFE_{ALGO} / AFE_{MGABC},$$

Here ALGO includes Basic ABC, MeABC and GABC. If  $AR > 1$  means MGABC converges faster. Table IV shows comparison between MGABC – ABC, MGABC – MeABC and MGABC – GABC. It is clear from Table IV that, for most of the test problems, convergence speed of MGABC is quicker amongst all the considered strategies.

TABLE IV ACCELERATION RATE (AR) OF MGABC COMPARE TO THE BASIC ABC, MEABC AND GABC

Test Problem	ABC	MeABC	GABC
Griewank	1.881305	1.554591	0.738034
Rastrigin	2.559262	2.509449	0.871124
Alpine	1.400583	1.401255	0.77677
Salomon Problem	5.409326	1.35365	3.975313
Inverted Cosine wave function	2.402306	1.717654	1.021312
Neumaier 3 Problem	4.008942	0.967447	4.002468
Beale function	5.608562	1.994998	3.439937
Colville function	3.410269	2.372872	3.17024
Kowalik function	2.029725	1.350409	1.645281
Shifted Rosenbrock	1.221816	1.171878	1.052992
Shifted Griewank	2.401641	1.5379	0.884583
Hosaki Problem	5.180235	0.853751	0.380367
Meyer and Roth Problem	5.274603	2.455203	0.969583
Sinusoidal	2.723964	1.897874	1.257644
Welded Beam Design Problem	1.619896	1.494782	1.395646
Compression Spring Problem	1.089133	1.046056	1.049892

**VI. CONCLUSION**

This paper proposed a novel approach for function optimization. The proposed Memetic search with global best solution in ABC (MGABC) is implemented in C programming language and tested over sixteen standard problems including welded beam design optimization problem and compression spring problem. Comparison of results shows that the newly proposed MGABC is better than basic ABC and its recent variants namely MeABC and GABC in terms of performance. Algorithms are compared on the basis of success rate and average function evaluation. It is implicit that if either success rate is higher or average number of function evaluation is less then algorithm is considered as good. Subsequently MGABC is compared on the basis of acceleration speed. Comparison of acceleration speed shows that MGABC is good in contrast to ABC, MeABC and GABC for 16, 14 and 8 problems respectively.

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